What seems to us of considerable interest biologically, is the extraordinary perfection, demonstrated in the figures here presented, of the self-regulation of population growth. Disturbances of the course of the birth rate have been compensated for, in the population and period under review, with the greatest nicety. It will be of interest to examine the facts for other populations, and this we hope later to do.

[1] Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health, Johns Hopkins University, No. 50.

[2] R. Pearl, "The Effect of the War on the Chief Factors of Population Change," *Science* (N. S.), **51,** 1920 (553–556).

[3] Pearl, "The Vitality of the Peoples of America," *Amer. J. Hyg.*, **1,** 1921, pp. 592–674.

[4] Pearl, "A Further Note on War and Population," *Science* (N. S.), **53,** 1921, pp. 120–121.

[5] *Quarterly Return of Marriages, Births, and Deaths Registered in England and Wales, etc.* Published by Authority of the Registrar-General. London. H. M. Stationery Office. *Passim.* Also corresponding Weekly Return, and Annual Reports.

[6] "For a Discussion of the Significance and Advantages of Plotting Trend Lines on a Logarithmic Scale," cf. Fisher, I. "The 'ratio' Chart for Plotting Statistics," *Q. P. Am. Stat. Ass.*, **15,** 1917, pp. 577–601, and Field, J. A., "Some Advantages of the Logarithmic Scale in Statistical Diagrams," *J. Pol. Econ.*, **25,** 1917, pp. 805–841.

[7] E. M. Elderton, "Report on the English Birth Rate. Part I. England North of the Humber." *Eugenics Lab. Mem.*, **19** and **20,** Cambridge Univ. Press, 1914, pp. viii and 246.

SEASONAL FLUCTUATIONS OF THE VITAL INDEX OF A POPULATION[1]

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In earlier papers the writer[2] has discussed various aspects of the birth-death ratio, or vital index of a population, including age and sex specific vital indices, racial and secular fluctuations, etc. Table 1 of the preceding paper by Pearl and Burger[3] gives the necessary data for finding, for one population, that of England and Wales, the course of the seasonal fluctuation of this important demographic constant. It is the purpose of this note to set forth biometrically the facts on this point, from the basic data mentioned.

Table 1 gives the frequency distributions for variation in the value of the vital index of the population of England and Wales in each of the four quarters of the year, ending, respectively, on March 31, June 30, September 30, and December 31, during the years 1838 to 1920, inclusive. The source of the data is given in Pearl and Burger already cited.

The significant biometric constants from table 1 are presented in table 2. It is apparent that there are a few highly aberrant observations in

TABLE 1

FREQUENCY DISTRIBUTION FOR VARIATION IN THE VITAL INDEX IN THE FOUR QUARTERS OF THE YEAR

VITAL INDEX	QUARTERS ENDING			
	MARCH 31	JUNE 30	SEPT. 30	DEC. 31
65– 69	—	—	—	—
70– 74	—	—	—	—
75– 79	1	—	—	—
80– 84	—	—	—	—
85– 89	—	—	—	—
90– 94	—	—	—	—
95– 99	—	—	—	—
100–104	—	—	1	—
105–109	1	—	—	—
110–114	—	—	—	—
115–119	3	—	—	—
120–124	3	—	—	1
125–129	1	—	—	1
130–134	3	2	—	4
135–139	9	3	3	4
140–144	11	2	—	7
145–149	13	2	3	5
150–154	9	7	5	9
155–159	13	6	5	11
160–164	10	5	10	9
165–169	2	11	9	17
170–174	3	9	8	7
175–179	—	11	8	4
180–184	—	9	6	2
185–189	—	10	3	—
190–194	—	2	4	—
195–199	1	1	8	1
200–204	—	2	1	—
205–209	—	1	3	—
210–214	—	—	1	—
215–219	—	—	4	—
245–249	—	—	1	—
Total	83	83	83	83

TABLE 2

BIOMETRIC CONSTANTS DEDUCED FROM TABLE 1

QUARTER ENDING	MEAN	STANDARD DEVIATION
March 31	147.06 ± 1.21	16.32 ± .85
June 30	169.89 ± 1.21	16.35 ± .86
September 30	175.43 ± 1.66	22.39 ± 1.17
December 31	156.46 ± 1.27	17.14 ± .90

the series, but it has not been thought best to discard or adjust them in this first treatment of the subject. Consequently it is to be understood that all the values given in table 1 were used in computing the values in table 2.

From these data the following points are to be noted:

1. The vital index has the lowest mean value in the quarter ending on March 31, the winter quarter. In that period the birth incidence is relatively low and the death incidence relatively high.

2. Next in value to this, but standing 9.40 ± 1.75 points above it, is the mean vital index for the autumn quarter ending December 31. The difference being 5.4 times its probable error, may be regarded as significant.

3. The spring quarter, ending June 30, shows the next higher mean, being 13.43 ± 1.75 points above the winter quarter.

4. The highest value of the index falls in the summer quarter, when births are most frequent and deaths least so. The mean value, however, lies only 5.54 ± 2.05 above that for the spring quarter, a difference which cannot be regarded as significant.

5. In variability of the vital index, the first two and the last quarters of the year, all exhibit significantly the same status. The vital index is distinctively more variable in the summer quarter, the difference in standard deviations when this quarter is compared with that ending June 30, amounting to 6.04 ± 1.45. This may be regarded as significant, being 4.2 times its probable error.

It thus appears that the extremely close compensatory relation between birth rate and death rate, which Pearl and Burger[3] have shown to hold in annual figures, does not obtain within the single year. Instead there is a well-marked statistically significant intra-annual, or seasonal fluctuation of the birth-death ratio.

[1] Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health, Johns Hopkins University, No. 51.

[2] Cf. particularly Pearl, R., "The Vitality of the Peoples of America," *Amer. J. Hyg.*, **1**, 1921 (592–674).

[3] Pearl, R. and Burger, M. H., "The Vital Index of the Population of England and Wales, 1838–1920," *Proc. Nat. Acad. Sci.*, **8**, 1922, pp. 71–76.

A SOLUTION OF THE LINEAR MATRIX EQUATION BY DOUBLE MULTIPLICATION

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1. *Methods of Sylvester and of Maclagan Wedderburn.*—A matrix A of order N is defined as a binary assemblage of N^2 elements a_{ik}, where either subscript may have any value from 1 to N. We add two matrices by adding corresponding elements. We multiply by the rule

$$(AB)_{ik} = \Sigma a_{is} b_{sk}; \ (s = 1, 2, \ldots, N). \qquad (1)$$

If $A_1, A_2, \ldots, A_h$ and $B_1, B_2, \ldots, B_h$ and C are all known matrices, while x is a required matrix, the linear matrix equation may be written